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Many-body effects in nuclear structure

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Abstract. We calculate, for the first time, the state-dependent pairing gap of a finite nucleus (^{120}Sn) diagonalizing the bare nucleon-nucleon potential (Argonne v_{14}) in a Hartree-Fock basis (with effective k-mass $m_k \approx 0.7m$), within the framework of the generalized Bogoliubov-Valatin approximation including scattering states up to 800 MeV above the Fermi energy to achieve convergence. The resulting gap accounts for about half of the experimental gap. The combined effect of the bare nucleon-nucleon potential and of the induced pairing interaction arising from the exchange of low-lying surface vibrations between nucleons moving in time-reversal states close to the Fermi energy accounts for the experimental gap.

PACS. 21.30.Fe Forces in hadronic systems and effective interactions – 21.60.Jz Hartree-Fock and random-phase approximations – 21.60.-n Nuclear structure models and methods – 27.60.+j $90 \leq A \leq 149$

At the basis of the theory of superconductivity proposed by Bardeen, Cooper and Schrieffer [1], one finds the concept of Cooper pair. This concept finds its origin in the solution of the problem, first studied by Cooper [2], of a pair of electrons interacting above a noninteracting Fermi sea of electrons. Thus, all but two of the electrons are assumed to be noninteracting. The background electrons enter the total problem only through the Pauli principle by blocking states below the Fermi surface from participating in the remaining two-particle problem. Cooper found that a bound state exists for arbitrarily weak coupling so long as the electron-electron potential is attractive near the Fermi surface. In the BCS wave function describing the ground state of the superconductor, pairs greatly overlap with each other in space, and it is the strong pair-pair correlations, in addition to the correlation between mates of a pair, which are ultimately responsible for the presence of an energy gap in the excitation spectrum of the superconductor. The apparent lack of dc electrical resistance, that is the existence of a critical temperature T_c below which metals superconduct, was first observed in 1911 [3]. It was not until almost four decades later that the basic forces responsible for the condensation were recognized [4]: the effective interaction between electrons arises from the exchange of crystal lattice vibrations (phonons). Strong support for this mechanism was found in the fact that T_c is larger for metals made out of lighter isotopes of a given element. In fact, if lattice vibrations were not important in the phenomenon of superconductivity there would be no reason why T_c should change as neutrons are added to or removed from the nuclei, since their main effect is to change the mass of the ions.

The nuclear structure exhibits many similarities with the electron structure of metals. In particular, most nuclei with an even number of neutrons and protons different from magic numbers, display an energy gap between the ground state and the first intrinsic (noncollective) excitation. The analogy between this gap and that observed in the excitation of superconducting metals first put forward by Bohr, Mottelson and Pines [5], led to the study of pairing correlations in nuclei. Following the original suggestion, BCS pairing theory was used to calculate the single-particle and collective-excitation spectra of atomic nuclei. The short-range interaction acting in the $^1S_0$ channel between pairs of nucleons moving in time-reversal states close to the Fermi energy, and responsible for nuclear condensation, has been parametrized in terms of a single constant ($G \approx 25/A$ MeV, A being the mass number), or in terms of effective interactions, the so-called Gogny force being one of the most successful [6]. One-particle transfer reactions measure the energy distribution and occupation number parameters, and reasonable agreement is obtained between the smeared Fermi surface characteristic of the pairing theory and these experiments. Two-particle

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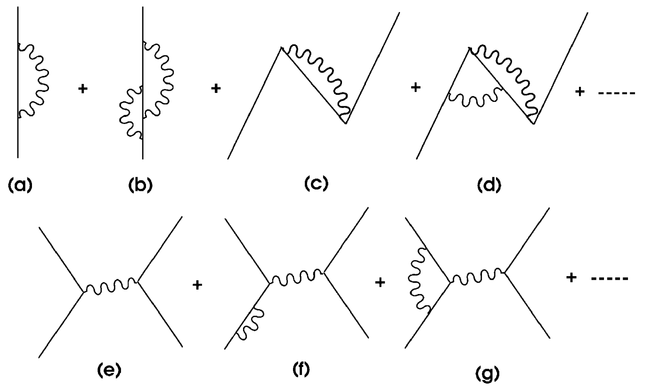

Fig. 1. Renormalization processes arising from the particle-vibration coupling phenomenon. The straight lines indicate quasiparticles obtained from BCS theory, making use of the mean-field single-particle states of Sly4 and the nucleon-nucleon v_{14} Argonne potential. The wavy lines indicate the vibrational states.

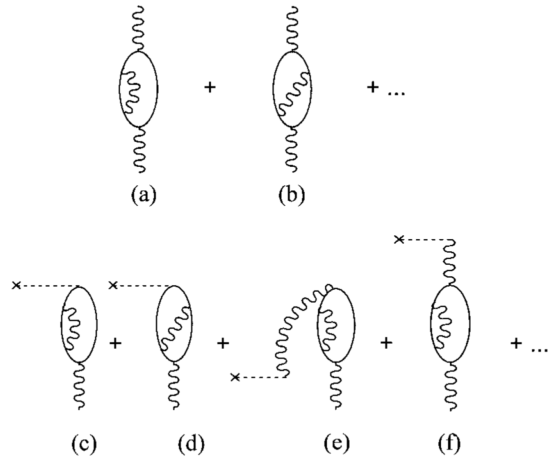

Fig. 2. Most relevant processes taken into account in the renormalization of the energy of the phonon (a-b) and of the associated transition strength (c-f).

transfer reactions [7] as well as odd-even mass differences [8] provide values of the experimental gap. Although theory is in overall agreement with these experimental findings, it is not able to account for the marked isotopic effects observed throughout the mass table, which is strongly correlated with the collectivity of the low-lying collective nuclear vibrations.

In fact, it has been recently found that a consistent fraction of the pairing gap (about half of it) is due to a long-range pairing force arising from the exchange of low-lying collective vibrations. It is this component of the pairing force which provides the correct isotopic dependence of the nuclear pairing gap [9]. In keeping with these results, one can posit that a quantitative description of pairing correlations in nuclei can be attained by correlating pairs of nucleons through the bare nucleon-nucleon potential and the exchange of collective surface vibrations [10]. This is demonstrated in the present paper for the case of typical superfluid nuclei, namely ^{119}Sn, ^{120}Sn and ^{121}Sn.

The formalism we shall use is based on the Dyson equation [11]. It can describe on equal footing the dressed one-particle state $\tilde{a}$ of an odd nucleon renormalized by the (collective) response of all the other nucleons (figs. 1(a)-(d)), the renormalization of the energy $\hbar\omega_\nu$ (figs. 2(a)-(b)) and of the transition probability $B(E\lambda)$ (figs. 2(c)-(f)) of the collective vibrations of the even system where the number of nucleons remains constant (correlated particle-hole excitations), and the induced interaction due to the exchange of collective vibrations between pairs of nucleons [9], moving in time-reversal states close to the Fermi energy (figs. 1(e)-(g)). We include both self-energy and vertex correction processes, thus satisfying Ward identities (cf., *e.g.*, [12]). Within this framework, the self-consistency existing between the dynamical deformations of the density and of the potential sustained by "screened" particle-vibrations coupling vertices leads to renormalization effects which make finite (stabilize) the collectivity and the self-interaction of the elementary modes of nuclear excitation, in particular of the low-lying surface vibrational modes, providing an accurate description of many seemingly unrelated experimental findings, in terms of very few (theoretically calculable) parameters, namely: the k-mass m_k [13] and the particle-vibration coupling vertex $h(ab\nu)$, associated to the process in which a quasiparticle changes its state of motion from the unperturbed quasiparticle state a to b, by absorbing or emitting a vibration ν [14].

The Dyson equation describing the renormalization of a quasiparticle a, due to this variety of couplings is

$$\left[\begin{pmatrix} E_a & 0 \\ 0 & -E_a \end{pmatrix} + \begin{pmatrix} \Sigma_{11}(\tilde{E}_a) & \Sigma_{12}(\tilde{E}_a) \\ \Sigma_{12}(\tilde{E}_a) & \Sigma_{22}(\tilde{E}_a) \end{pmatrix}\right] \begin{pmatrix} \tilde{x}_a \\ \tilde{y}_a \end{pmatrix} = \tilde{E}_a \begin{pmatrix} \tilde{x}_a \\ \tilde{y}_a \end{pmatrix}, \quad (1)$$

where Σ_{ii} and $\Sigma_{ij}, (i \neq j)$ are the normal and abnormal self-energies. The quantities E_a denote the quasiparticle energies obtained from a previous diagonalization of the bare nucleon-nucleon potential within the framework of the generalized Bogoliubov-Valatin transformation (that is, the extended BCS calculation, which includes pairs of particles with different number of nodes in the Cooper pair wave function).

Equation (1) is to be solved iteratively, and simultaneously for all the involved quasiparticle states. At each iteration step, the original quasiparticle states a with occupation numbers u_a and v_a and quasiparticle energies E_a, become fragmented over the different eigenstates $\tilde{a}$ with probability $\tilde{u}_a^2 + \tilde{v}_a^2$, while the renormalized occupation factors are obtained from the components of the eigenvectors, $\tilde{x}_a$ and $\tilde{y}_a$, according to the relations $\tilde{u}_a = \tilde{x}_a u_a + \tilde{y}_a v_a$, $\tilde{v}_a = -\tilde{y}_a u_a + \tilde{x}_a v_a$. The quantities $\tilde{u}_a$ and $\tilde{v}_a$ are related to the spectroscopic factors measured in one-nucleon stripping and pick-up reactions, respectively. One can also define [11,12] a renormalized state-dependent pairing gap, through the relation $\tilde{\Delta}_a = 2\tilde{E}_a \tilde{u}_a \tilde{v}_a / (\tilde{u}_a^2 + \tilde{v}_a^2)$, which in the limit of no fragmentation reduces to the usual BCS expression [15].

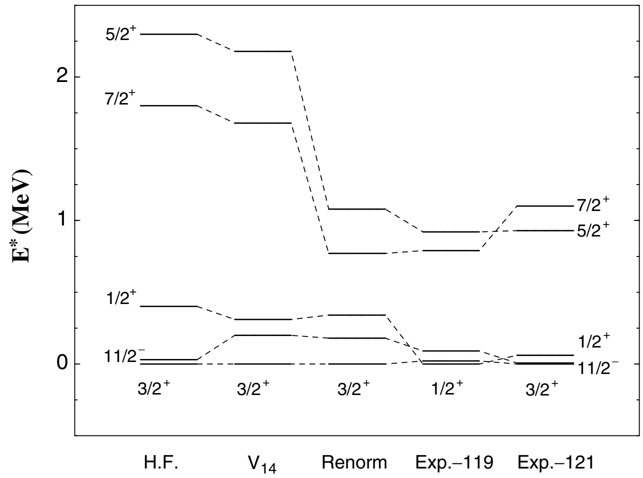

Fig. 3. The spectra of the lowest quasiparticle states in ^{120}Sn calculated using Hartree-Fock theory, BCS with the Argonne v_{14} potential, and after renormalization, are compared to the experimental levels in the odd neighbouring nuclei ^{119}Sn and ^{121}Sn.

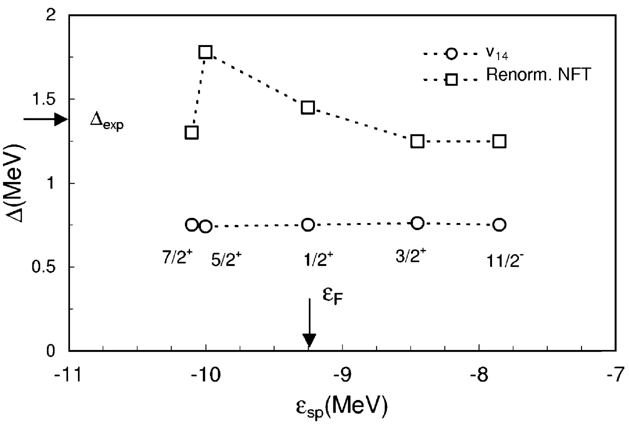

Fig. 4. The state-dependent pairing gap for the levels close to the Fermi energy obtained using BCS theory with the v_{14} Argonne potential (circles) is compared with the result obtained including renormalization effects (squares).

In the calculations reported below, a Skyrme interaction (Sly4 parametrization, with $m_k \approx 0.7m$ [17]), was solely used to determine the properties of the bare single-particle states and the collective vibrations in the particle-hole channel. On the other hand, in the particle-particle (pairing) channel the interactions used were the bare nucleon-nucleon v_{14} Argonne potential and the exchange of collective vibrations.

As seen from fig. 3, Hartree-Fock theory is not able to account for the experimental quasiparticle energies of the low-lying states. Diagonalizing the Argonne v_{14} nucleon-nucleon potential in the Hartree-Fock basis, within the framework of the generalized Bogoliubov-Valatin approximation including scattering states up to 800 MeV above the Fermi energy (to achieve convergence) in a spherical box of radius equal to 15 fm, one obtains the state-dependent pairing gap shown in fig. 4 (labelled v_{14}). The

Table 1. The energy and reduced $E2$ transition strength of the low-lying 2^+ state, calculated according to different theoretical models, are compared to the experimental values [27].

	$\hbar\omega_{2^+}$ (MeV)	$B(E2\uparrow)$ (e^2 fm^4)
RPA (Gogny)	2.9	660
RPA (Sly4)	1.5	890
RPA + renorm. [23]	0.9	2150
Exp.	1.2	2030

resulting pairing gap (average value for levels around the Fermi energy) accounts for about half of the empirical pairing gap value ($\approx$ 1.4 MeV) obtained from the odd-even mass difference [18]. In keeping with this result, the quasiparticle spectrum (cf. fig. 3), although being slightly closer to the experimental findings than that predicted by Hartree-Fock theory, displays large discrepancies with observations. The situation is rather similar concerning the low-lying quadrupole vibration of ^{120}Sn calculated in the QRPA with standard effective nucleon-nucleon interactions like Gogny or Skyrme forces. While energy is predicted too high, which may not be too important, the $B(E2)$ value is a factor 2-3 too small (cf. table 1), a result which calls for a better theory.

In fact, renormalizing the energy and the transition strength of the 2^+ phonon, following Nuclear Field Theory [20,21], that is, considering the couplings of the type depicted in fig. 2 (cf. also ref. [22] and references therein), one obtains an increase of the $B(E2)$ transition probability which brings theory essentially in agreement with experiment (cf. table 1) [23]. The most important processes which renormalize the energy of the phonon are shown in figs. 2(a) and (b). While these two contributions tend to cancel each other in a normal system, this is not the case in a superfluid nucleus. In fact the phonons are calculated in a Bogoliubov-Valatin-quasiparticle basis, and while the cancellation is strong in the particle-hole channel, the opposite is true in the particle-particle channel [25]. Other graphs which are also of fourth order in the particle-vibration coupling vertex, but contain intermediate states with more than four quasiparticle states, lead to very small contributions. This is because these terms not only involve larger denominators, but also, due to their higher degree of complexity, give rise to contributions with "random" signs which tend to cancel each other. In keeping with the above discussion, the most important processes renormalizing the $B(E2)$ transition probability are those shown in figs. 2(c), (d), (e) and (f).

We have also calculated the static quadrupole moment Q of the 2^+ state, including the contributions from the processes shown in fig. 6.27 of ref. [14], considering also self-energy effects [26]. The resulting value of Q is rather small (8 e fm^2), in agreement with the experimental findings (10 ± 10 e fm^2, or -5 ± 10 e fm^2 [27,28]).

Because in the above calculations we have included only a partial set (although the most important for the physics under discussion) of the NFT graphs needed to provide a completely consistent description of single-particle and collective-vibration renormalizations, the mixing of spurious states with the physical states has to be contemplated. Although it is difficult to give a precise estimate of the error induced by such undesired couplings, 30% effects have been found in the calculation of the energy of the one-phonon state [29].

Making use of phonons which account for the experimental findings, the normal and abnormal self-energies were calculated, and eq. (1) solved. The average value of the resulting state-dependent pairing gap of ^{120}Sn is now close to the value $\Delta_{\rm exp} = 1.4$ MeV derived from the odd-even mass difference (cf. fig. 4). In fig. 3 we show the energy of the peaks carrying the largest quasiparticle strength, for the orbitals around the Fermi energy, which provide an overall account of the lowest quasiparticle states measured in the odd systems ^{119}Sn and ^{121}Sn.

One can conclude that mean-field theory and bare nucleon-nucleon potentials reproduce neither the experimental transition strengths nor the pairing gaps, let alone the density of quasiparticle states close to the ground state. Dressing the single-particle motion, the correlated particle-hole excitations of mean field and the nucleon-nucleon interaction with collective surface vibrations, brings theory in overall agreeement with experiment. In particular, about half of the pairing gap arises from the long-range component of the pairing interaction associated with the exchange of collective vibrations. To further clarify the interdependence of single-particle and collective degrees of freedom, future studies should, for example, concentrate on the role this interdependence has on the nuclear masses. In particular, whether the explicit, simplified, inclusion of ground-state correlations and of the induced pairing interaction can reduce the present r.m.s. error of 0.674 MeV with which one of the best presently available Hartree-Fock mass formula [30] is able to reproduce the experimental findings.

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